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On the nature of bilogic: the work of Ignacio Matte Blanco

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1. Introduction

1.1 Overview

The psychologist Ignacio Matte Blanco is known for his development over many years of the thesis that part of the human mind behaves as if it is governed by a type of logic that is different from conventional logic. The interplay of this new logic and conventional logic results in a bizarre combination that he called *bilogic*. I will very briefly describe the essence of this by an idealised example.

Suppose I have a dream in which a friend of mine, let us call him Peter, turns into a dragon. Conventionally, an analysis of this dream would proceed by *association*: I, or a therapist, would elicit associations with the elements of the dream, which might include the idea that "dragons are dangerous" and this might raise to consciousness my feeling that Peter was in some way dangerous. Matte Blanco's suggestion is that, rather than thinking just in terms of the rather weak link of association, we see the process of the generation of the dream as proceeding *as if* there were taking place a quasi-logical sequence of propositions, of the form

1. Peter is dangerous
2. Dragons are dangerous things
3. Dangerous things are dragons
4. (From 1 and 3) Peter is a dragon

The passage from 2 to 3 is (conventionally) a *non sequitur* because "are" denotes the relation of sub-class, and this is not a symmetric relation: a statement of the form "*A*s are *B*s" does not necessarily imply the statement "*B*s are *A*s". Matte Blanco's central postulate is that the Unconscious treats *all* relations as symmetric. From this postulate, a wide range of consequences flows, which unifies a great many aspects of unconscious processes.

So far this idea has attracted only a small following, because it seems to fly in the face of all that one normally associates with the term "logic". The normal reaction to the "deduction" expressed described above is to declare that it is *not* logical. The aim of this paper is to contribute to a program of understanding Matte Blanco's theory in logical terms, and in particular to enable bilogic now to take its place in the family of non-classical logics, including quantum logic and intuitionism.

In fulfilling the above aim, I draw strongly on the work of Bomford (1999), which renders Matte Blanco's system much more coherent. Bomford notes that Matte Blanco starts from two disparate principles: that of symmetry, expressed in terms of logic, and that of generalization, expressed in terms of set theory (the prinicple that anything is considered as though it were equivalent to a class of which it is a member). While staying close to Matte Blanco's content, Bomford reformulates the theory on the basis of logic alone, and also distinguishes the purely logical (or quasi-logical) structure from its subsequent applications in psychology and elsewhere. I shall follow this approach, particularly with regard to freeing the theory from its reliance on set theory. The mathematics used by Matte Blanco was understandably dominated by the set theory that was in vogue when he started work. Now, however, his insights can be reformulated to take account of later approaches to logic.

The plan of this paper is as follows. Section 2 is an exposition and critique of the core ideas of Matte Blanco as they concern the logic of the Unconscious. The aim here is to understand what aspects of the theory will be key to its subsequent application in psychology, and thereby to formulate the theory itself so as to make this application as effective as possible. The section is thus divided into two parts dealing with his concept of the Unconscious and with the core logical properties that he thinks are required in order to capture the way the Unconscious appears to act. Section 3 introduces an alternative way of looking at the duality between symmetric logic and asymmetric logic, from a different psychological perspective, which in places will allow a wider insight into what is going on. Section 4 then returns to Matte Blanco to consider perhaps the most important consequence of the introduction of bilogic, the dominant role of the concept of infinity. Finally section 5 formalises the discussion into a revised conception of the nature of bilogic.

For those acquainted with Bomford's work, I shall itemise here the points where I add to it or propose alternatives.

1. I follow Bomford in seeing the division between symmetric and asymmetric logic (which gives rise to the characterisation of the system as a "bilogic" – given formal expression section 5) as fundamental.
2. I criticise (section 2.2.2) the deduction of the principle of generalisation from the principle of symmetry, deriving this principle instead from a reformulation of the basic logic using a different logical connective.
3. I place Bomford's law of rotation at the level of syntax rather than inference, together with the law of symmetry as applied to two-term propositions other than those asserting membership or subclass (section 2.2.4). This is largely a matter of convenience rather than real content.
4. I add a development (section 4) of the role of infinity in the theory, based on the concept of order rather than Matte Blanco's concept of cardinality, which I believe is truer to the psychological data
5. I do not adopt Bomford's law of negation as universally valid (I think this is psychologically unwarranted – see section 2.2.3) but instead deduce a limited form of it from the above concept of infinity (section 4.2.2)
6. While agreeing that the system is not a logic in the usual sense, I argue that it can be given a completely rigorous mathematical formulation, which I characterise as a "context dependent logic". The concept of context is developed in section 2.2.1 and that of context dependent logic in 5.2
7. I further extend the range of psychological applications of the theory by linking it with modern experimental psychology as well as analytical psychology (section 3).

1.2 Introductory remarks regarding logic in general

Two questions arise at the outset: first, does this model ("model" in the sense of something related to observations by an "as if") deliver insights which justify its greater complexity than a simple theory of association? and second, is it legitimate to call it a logic? The first is a matter of personal judgement, to be reached after studying the literature of bilogic. The second needs to be addressed here, at least briefly, because many readers will object to the word "logic" being applied to a system that is manifestly inconsistent, and thus "illogical".

The question of what logic is, and what it is relevant to, is contested. I follow Bomford in separating the *content* of logic, as an intellectual domain, from its subsequent application. The first has developed through the system of syllogisms of mediaeval times, through the mathematical versions of Leibniz and Boole, and the failed attempt at a grand universal system by Russell and Whitehead, to the diversity of modal logics, tensed logics, quantum logics and so on available today. The *application* of logic has been viewed either as descriptive of how people in fact think, or as prescriptive of how people ought to think if they are to reach correct conclusions. From the second point of view, bilogic is not a logic since its conclusions are not "correct" in the normal sense. We are here not thinking of logic as prescriptive, but as descriptive, as a model for the generative processes underlying thought. Logic in this sense is thought of, not as strictly governing these processes, but as a set of conceptual principles to shed light on what was happening. We now have a far wider range of formal logics, and a far wider range of psychological models, than was available to Matter Blanco. We can thus set about re-examining his arguments so as to choose a form of logic that best expresses his fundamental insights and sheds most light on the phenomena of human existence. This is my aim in what follows.

2. Matte Blanco's description of the Unconscious

2.1 The term "Unconscious"

Matte Blanco takes considerable care to articulate a distinction between, on the one hand, the (Freudian) Unconscious (which, for clarity, I will spell with a capital U, though he does not do so), and on the other hand the categories of conscious/consciousness, and unconscious/unconsciousness. In his work (at least in the earlier sections) "consciousness" refers, at its most general usage, to propositional thought characterised by focussed awareness. Thus "consciousness … cannot focus on more than one thing at a time" (p 97) and consciousness "is by its nature analytical" (p 111). But there is a division within this category. Citing Spearman, he states that

> "[t]here is a double aspect in the nature of conscious knowledge… We may … keep the important distinction which [Spearman] makes of the two aspects of thinking or *propositional* activity or establishment of relations …: one, the establishment of relations which concern an object (which can, among others, be ourselves or some aspect of our inner life) and the other, a second establishment of relations which refers to the first and by means of which a content enters consciousness as consciousness of a content (p 108, my emphasis)."

Thus while, for Matte Blanco, all consciousness is propositional, this second mode of consciousness, of "knowing that we know", is in addition reflexive. It is also concerned with description, as opposed to simple experience:

> "Consciousness … looks upon the (infinite) base [of the Unconscious] and makes attempts at describing it. But the *experience* of being cannot be described" (p 101, MB's emphasis).

Though he is never totally explicit about it, Matte Blanco also seems to assume that consciousness, at least in this reflexive mode, is verbal (cf. pp 114-5).[1]

This propositional conception of consciousness seems to have been very widespread in much early psychological literature. When comparing with modern work, however, it is important to realise that modern authors occupy a spectrum of definitions between Matte Blanco's focussed, reflexive, verbal thought at one extreme (e.g. Daniel Dennett) and simple awareness at the other (e.g. Max Velmans). This could reflect a difference in the actual balance of prominence within the content of these authors' own awareness, coupled with a tendency to assume that this is uniform over all human beings. Intellectuals tend to talk to themselves a lot, unless they have trained themselves to examine and control this, and so regard it as normative for human consciousness.

To return to Matte Blanco, it follows from his narrow definition of consciousness that when he refers to things that are for us "unconscious", or to "unconsciousness" (which he distinguishes on p 150 from the Unconscious), we should not assume that he is referring to things of which we are unaware, or which are not part of our experience.

When we come to the Unconscious, however, we are very explicitly dealing with a specific internal system, as postulated by Freud and as explored through psychoanalytic technique. It is, as part of this theoretical scheme, unconscious (with a small u) and probably also, in Matte Blanco's view, absent from our awareness, though the above reference to "Consciousness … looks upon the (infinite) base" might suggest that aspects of the Unconscious might be part of our experience or awareness. Matte Blanco spends considerable effort in examining the Freudian concept of the Unconscious, and he adopts a broad version of it, as the location of primary process and as the foundation of all experience, which he argues is in accord with ideas that Freud *de facto* maintained throughout his work, even when the terminology was altered later in Freud's development. It is to this broad theoretical internal system, that the non-standard logic is claimed to apply.

2.2 The logical ideas used by Matte Blanco

2.2.1 Basic ideas

In part II of Matte Blanco's book the main logical structure are explained in terms of set theory. At this point he refers to the Unconscious, in the sense discussed above, as "the system Ucs."

The first principle is

> I. The system Ucs. treats an individual thing (person, object, concept) as if it were a member or element of a set of class which contains other members; it treats this set or class as a subclass of a more general class, and this more general class as a subclass or subset of a still more general class, and so on.

(Note that Matte Blanco, as he explains later, is not in fact attaching any significance here to the distinction between "class" and "set", of which he is aware.) Two points immediately emerge from this formulation.

The importance of context

Whereas in abstract mathematics relationships such as membership of classes are considered as eternal verities all simultaneously coexisting, in "the system Ucs" each thing is seen on a particular occasion as contained in a particular nested series of classes, which provides the context for that thing on this occasion. There will be a range of other classes of which it might be considered a member, but these are for the time being out of the picture. We need to know what is the implicit nested series, the context, before we can understand the logical processes being executed. This motivates me to look for a *context dependent logic* in order to understand what is going on.

The distinction between membership and subclass

Matte Blanco in this principle distinguishes membership (in the first clause) from being a subclass/subset (in the later clauses). He has in mind the conventional picture of classical logic in which things are either individuals or classes (species, genera), with the classes being defined in terms of some properties of the individuals constituting it. The individual is a *member* of various classes, whereas classes of increasing generality are *subclasses* of each other. I will return later to discuss the validity of this picture.

2.2.2 Symmetry and its consequences

The second principle is the *principle of symmetry* :

> II. The system Ucs. treats the converse of any relation as identical with the relation. In other words, it treats asymmetrical relations as if they were symmetrical.

from this he deduces a number of consequences, including

> II_2. When the principle of symmetry is applied the (proper) part is necessarily identical to the whole.

To quote the example whereby Matte Blanco explains this, given that "the arm is part of the body", the principle of symmetry implies that "the body is part of the arm". In general form, given that "*A* is a part of B" then "*B* is a part of A". If "part" here means "subset", then we have that *A* is a subset of *B* and *B* is a subset of *A*, from which, in ordinary set theory, it follows that *A* and *B* are identical. Thus the part is identical to the whole, and in the case of the particular example, the arm and the body are the same.

The process of argument here involves, quite deliberately, an alternation of deduction within ordinary logic (such as set theory) and deduction within a structurally different “symmetric logic”. This interplay of the two systems leads Matte Blanco to call the structure here a “bilogic”. I shall return to this aspect later. First, however, I want to examine an example that he draws from the preceding principle, which is essentially the vital “principle of generalisation”.

> $II_{2\,a}$ When the principle of symmetry is applied, all members of a set or of a class are treated as identical to one another and to the whole set or class …

In this instance “part” in principle II_2 has become “member”. Then II_2 has been invoked to deduce that the member is identical to the class of which it is a part, and hence that all members of the class are identical. The trouble with this is that the set theory part of the deduction of II_2 from II (which I have sketched above after the statement of II_2) is *not* valid in the case where *A* is a *member* of *B*, as opposed to a *subset* of *B*. Note that the same objection holds in the case of Bomford’s formulation “For if A is a member of a class B, then B is a member of A, and so anything is considered as though it were equivalent to the class of which it is a member.”

Let us spell out the attempted bilogic deduction of II_2 from II in this case.

1. It is given that *A* is a member of *B*
2. From II, therefore, *B* is a member of *A*

Now the argument can take two courses. The first course follows conventional set theory where one imposes an additional axiom, the axiom of foundation [2]. It can easily be shown from this that the combination of 1 and 2 above is impossible. Instead, therefore, of deducing, as we want to, that *A* and *B* are identical, we deduce that they do not exist.

The second possible course is when one does not impose this axiom. Then no contradiction arises. In this case, however, one can find mathematical examples where the situation holds, but *A* and *B* are *not* identical. So in neither of these versions of set theory does one get the desired conclusion, namely $II_{2\,a}$.

This is a major difficulty, because the conclusion $II_{2\,a}$ is a central part of Matte Blanco’s whole work. As he expresses it later, “The individual does not *stand* for the class but, in contrast, class and individual … are one and the same thing” (p 171, MB’s emphasis). The failure of the deduction of $II_{2\,a}$ to be valid in set theory is the first indication that set theory is not the right system for the sort of bilogic that we are seeking. In particular, the distinction between “subset” and “member”, crucial in set theory, is much too refined for the Unconscious [3], and we need a more concrete sort of relation that does not distinguish these two in order to capture what is happening here. I introduce this relation in section 5.3.2.

2.2.3 The question of negation

A second sub-sub-case of the principle of symmetry is formulated as follows.

> $II_{2\,b}$ When the principle of symmetry is applied, certain classes whose propositional functions are of the type *p* and not-*p* ($p. \sim p$) and which, therefore, are empty by definition, may be treated as not empty.

A word of explanation may be required here. Matte Blanco is primarily interested in classes which are defined by a specification of the form

> Every entity *x* for which *p*

where “*p*” stands for some proposition involving the symbol *x*; for example

> *p* = “*x* is my sibling”

(in which case the class in question is the set of all my brothers and sisters, if any). A proposition involving a symbol (conventionally *x*) which is used in this way in the definition of a class was called by Russell and Whitehead and their contemporaries a "propositional function." (The conventions are slightly different today). The propositional function represents the class, but it is more specific that the class: a given class can be described in many different ways, by different propositional functions.

Now it would appear at first sight that $II_{2\ b}$ is referring to propositional functions such as

x is my sibling and *x* is not my sibling

But a reading of the text indicates that this is *not* the sort of proposition that Matte Blanco is talking about. Rather, he is (to take his own example) considering propositions of the form

x is alive and *x* is dead (*)

If he were being really precise, propositions of this form would be formally expressed as "propositional functions of the type '*p* & *q*' where *q* implies ~*p*' ". The point here is that the proposition (*) does not use the word "not" (or its symbol), so that we are not talking about negation as such, but rather the *coincidence of contraries*.

In practice, Matter Blanco handles (*) by remarking that "being alive and being dead are subclasses of a higher class whose propositional function may include all possibilities regarding life"; II_2.is then applied to argue that these subclasses are identical, so that (*) is admissible. But the argument does not necessarily involve simple negation: this higher class might (following the imagination of the creators of the adventure games my son played when young) include the subclass of the "undead" such as zombies and vampires which are neither alive nor dead, in which case "alive" and "dead" would not be precise negations of each other, but simply varieties of life.

Matte Blanco, in my opinion, confuses the whole issue by using the phrase "*p* and ~*p*" as a shorthand for "*p* & *q* where *q* implies ~*p*". What he is in fact establishing is "the absence of the principle of contradiction", for which there is clear observational evidence in psychology. To make matters even more confusing, he links this absence with Freud's formulation that "there [is] … no negation" (p 46), and uses the phrase "the absence of negation" as equivalent to this, despite the fact that he repeatedly uses the symbol "~" suggesting that there *is* such an operation as negation!

I suggest that the use of the symbol "~" in describing the processes of the Unconscious is unwarranted by the evidence, and that the concept of global negation (that is, everything in the entire universe that is not *p*) is an abstraction that is, on Matte Blanco's own evidence, absent from the Unconscious. This system only recognises specific contraries, not global negations. In the Unconscious there is no negation, and there is no principle of contradiction. The first is an empirical fact, the second is a deduction from II_2 .

The situation is quite different when it comes to partially ordered classes, where the members possess some particular quality. In this case, as we shall see in section 4.2.2 below, we have not only a principle of non-contradiction, but we can *deduce* a "law of negation", as Bomford terms it, in which one can actually assert the negation of the proposition of possessing this quality. Depending on the context, the qualities "alive" and "dead" could alternatively be seen as instances of this, and hence subject to a law of negation.

2.2.4 Relations in general

The basis of the system being considered is the notion that relations become symmetrical. A fundamental objection might then be raised, however, namely that the very concept of "relation" appears to require an ordering of the terms in the relation, and thus a basic asymmetry. This leads Matte Blanco into a very delicate examination of the nature of relation and of logic (pp 322 ff). His discussion is founded firmly on conventional predicate calculus [4], and thus inherently self-limited, though he boldly transcends this limitation. In the course of his discussion he raises aspects of the content of bilogic that are of great interest, but which need not necessarily affect the formal way in which that logic is handled. I will here follow a different course, working from a more general logical context.

Logic tends to be based on a *formal language* – essentially, a formalised conception of marks placed in sequence on a piece of paper, related in its origin to sounds uttered in sequence in time. It is thus, in its normal form, rooted in a directional linear sequence, which is a thoroughly asymmetrical concept. The formal language is then used to represent (model) actual things or processes in the world, and in this representation some aspects of the language might be regarded as irrelevant scaffolding, and other aspects as the parts that correspond to actuality. This gives us three possible ways of overcoming the contradiction between an asymmetrical (linearly ordered) formal language and the symmetric subject mater that it is supposed to represent.

1. We could, as is customary, regard the elements of a formal language as being not actual marks on a piece of paper, but idealised marks conceived by identifying equivalent actual marks. Thus x, *x*, **x**, x and so on are all identified as being "lower case letter x". We could then use this distinction between actual marks and an idealised element that they represent in order to introduce symmetry into the formal language: we could simply *identify* certain collections of marks on paper that differed only in their arrangement. An implementation of this might be to identify linear sequences that differed only in their order, so that, for instance, the mark "(a, b, c)" was regarded as *the same mark as* "(c, b, a)".
2. As a second alternative, we could regard certain aspects of the ordering of symbols as part of the syntactic scaffolding having no relation to the semantics of the language. For example, "P(a, b, c)" and "P(c, b, a)" might be defined as different as regards the rules of writing and manipulating the language, but the semantics of the language might be required to be such that the truth-values of these strings were always the same.
3. The third alternative[5] would be to arrange that the rules of inference in the language were such that one was always allowed to deduce, from a given statement, any other statement that differed only in the order of certain elements. For instance, from "P(a, b, c)" one might always deduce "P(c, b, a)".

Since the aim here is to try to model actuality as closely as possible, I shall adopt the version 1, which is stronger than 2 or 3. In that case the assertion that three items *a, b* and *c* stood in a relation *R* would be written as "*R*(*a, b, c*)", which is the same mark as "*R*(*c, b, a*)". Thus relations (when written in this way) are necessarily symmetric.

2.2.5 Me

In a sense, the Self is the context for all propositions arising from the Unconscious, whether or not "me" enters as an explicit term, or is explicitly symbolised[6]. This constant implicit presence of "me" leads to an ambiguity when "me" could be a term in a relation. For example, the statement "John is a Brother" could have, as its essential content, either "John is someone who has a Brother", or "John is my Brother" (invoking the implicit constant presence of the "me" in the picture), or "John is a brotherly person" – or, and most likely, a combination of these with no real distinction being made between them. In order to allow for this fluidity, I will allow the rather non-standard concept of a 1-term relation which can, in appropriate contexts, be equivalent to a 2-term relation obtained by adding "me". I will explain this more fully later.

3. The "bi" in bilogic

Matte Blanco's approach is set firmly within the psychoanalytic context, although he recognises some aspects of experimental psychology, aspects which now seem rather outdated. In broad terms, symmetric logic operates within the Freudian Unconscious, and asymmetric logic within conscious thinking. A considerable part of his work is concerned with refining this formulation, and relating it to different approaches taken by Freud throughout his life, but I will not explore this here.

It would appear, however, that we are concerned with something more basic than a narrow interpretation of the Freudian Unconscious. In view of the fact that Matte Blanco stresses that he is not restricting his Unconscious to the repressed Unconscious, it is at least plausible to extend the idea to the whole of the mind that is not conscious, in the sense in which Matte Blanco uses this word – which, as we have seen, can include aspects that we are aware of. If that were the case, then we would appear to be dealing with two fundamentally different modes of operation of the human brain. We should expect, then, that the existence of these modes would be revealed by all forms of enquiry that recognised the existence of non-conscious processes, whether or not they worked within a psychoanalytic model.

That this indeed happens is evidenced most helpfully by the work of Teasdale and Barnard (1993) on Interacting Cognitive Subsystems (ICS)[7]. They start from experimental evidence for different forms of coding information; for instance, (a) immediate and sensory based, (b) verbal and logically based, or (c) a more holistic, meaning based coding. These and other distinct codes form the basis for nine postulated subsystems; three are sensory and proprioceptive, two involve higher order pattern recognition; two, the production of response, and two are yet higher order, meaning based systems. It is these last two, called the propositional and the implicational systems, which are of direct relevance to Matte Blanco. Memory is integral to each subsystem, and likewise distinguished by separate codes. Thus, the logical, propositional, memory is verbally coded, whereas the implicational memory, that records meaning at a more generic level, is encoded in a rich variety of sensory modalities, and is therefore more immediate and vivid. It also connects to the body's arousal system. It is concerned with overall meaning, with threat and with value and the status of the self. In Teasdale and Barnard's approach there is no overall arbiter between these two. They have to communicate constantly with each other for the whole thing to work. Clinical experience has suggested to many workers that failure of this communication between systems leads to the host of internal dissonances experienced by human beings as emotional problems, etc.

As I Clarke writes,

> a simple (probably far too simplistic!) way of understanding the dichotomy between the two ways of experiencing the world in interacting cognitive subsystem terms could proceed as follows: perhaps the everyday, scientific state is one where the propositional and implicational subsystems are working nicely together in balance, whereas the spiritual/psychotic state is one where the two are disjoint, and the system is essentially driven by the implicational subsystem. (2001, p 136)

She goes on to draw out the contrast at the logical level between these two modes:

> In both psychotic material, and accounts of spiritual experience and religious ideas from different faiths, a parallel logic can be discerned that is strikingly different from, and frequently opposite to, common-sense and scientific logic. … The logic of science has unlocked so many secrets that we have downgraded that whole area of human experience that lies beyond the reach of this logic. I suggest that this area has its own logic, which is even more compelling, but more difficult to pin down.
>
> It is the logic of archetypes; of myths and stories - full of paradox and a sense of mystery. Science discriminates - things are "either - or". In this realm two contradictory things can be simultaneously valid - a world of "both - and". In archetypal stories, such as the traditional fairy tales, the ordinary transitions of every human life, like leaving home and "seeking your fortune", falling in love and marrying, and death, are vested with cosmic significance. In Christianity, and indeed most religious traditions, the individual is both supremely precious, and insignificant in the context of the whole; God is distant and transcendent, and simultaneously concerned for the individual. (2001, p 138)

This line of argument indicates that a shift in the balance between the two principal cognitive subsystems can underlie experiences of a spiritual nature, which Bomford has linked with bilogic. In all this material, what we see is that the two components of bilogic, the symmetric and the asymmetric, arise from, respectively, the implicational and propositional subsystems, and that these combine with precisely the characteristics analysed by Matte Blanco. His analysis is thus strongly confirmed by the evidence, from a completely different methodology, that the symmetric and asymmetric components arise from two basic cognitive subsystems in the human mind.

4. Infinity

As indicated by the title of his main work , infinity is a vital part of the symmetric logic conception of the unconscious. Matte Blanco's handling of this is, however, made difficult by the wide range of different mathematical concepts which are all called “infinity”. I will begin by reviewing these, basing the discussion on conventional set theory. Of course, the question of infinity will then be further complicated because of the problems of the relevance of set theory that I have already raised! In section 4.1 I will describe the available range of notions of infinity in mathematics, and in section 4.2 I will consider the application of these to bilogic.

4.1 Infinity in mathematics

There are (at least) three different concepts of number in mathematics – cardinal, ordinal and arithmetic numbers, where I use the last term to embrace all the different varieties of “practical” numbers (natural, integers, rational, real, complex, quaternion, octonion, Cayley).

Cardinal numbers indicate *how many* elements there are in a set by establishing a one-to-one correspondence between the elements of the set and the elements of some standard reference set (for which different systems are available). The cardinal numbers *are* these standard reference sets. The cardinal number whose elements can pair up with those of a given set is called the *cardinality* (or power) of that set.

Ordinal numbers are associated with the conceptually different operation of “lining up” the elements in sequence and counting them – in the original form of the idea, uttering the words “one”, “two” and so on. Many non-human mammals and birds can be trained to estimate cardinals accurately up to about 7 (depending on the species), though they are not capable of performing the counting operation of ordinals, which seems to require language.

For the more “practical” *arithmetic numbers* the focus is not on set theory but on calculation. These start off with the set *N* of *natural numbers* 0, 1, 2, … . Then further numbers are hypothesised and added by processes that might be summarised as *replication, interpolation* and *extrapolation.* Replication produces the *negative numbers* (a second copy of the natural numbers, but regarded as going “backwards”); interpolation produces fractions (called *rational numbers*) and all limits of fractions (called *real numbers*); replication in a different sense produces *complex* and various other higher-dimensional sorts of numbers; and extrapolation produces ideal infinite numbers. All these operations are a matter of deciding on what rules the new numbers are to obey, as a matter of convenience.

The meaning of “infinite numbers” depends on which sort of numbers one is talking about. At the level of cardinal numbers, a set is called *finite* if every subset that is not equal to the whole (called a *proper* subset) has a smaller cardinality (a definition that Matte Blanco attributes to Dedekind). To understand how this might *not* happen (in which case the set is by definition *infinite*), consider the set $\omega = \{0, 1, 2, \ldots\}$ of all finite natural numbers, and out of this select the subset $E = \{0, 2, 4, ..\}$ of all even finite numbers. We can pair off the elements of w against the elements of *E* according to the scheme

$$0 \leftrightarrow 0,\quad 1 \leftrightarrow 2,\quad 2 \leftrightarrow 4,\quad 3 \leftrightarrow 6,\quad \ldots$$

So w and *E* have the same cardinality, even though *E* is a proper subset of ω; and therefore these sets are termed infinite.

Matte Blanco’s interest is expressed mainly in terms of cardinal numbers and the definition of infinite sets, which recalls property II_2 above. Though in set theory the part is not *equal* to the whole, as it is in symmetric logic, for infinite sets some of the parts are *equal in cardinality* to the whole. It is important to note, however, that cardinals are not the only sort of infinite number. Each sort of number has its own infinite versions. The cardinal number of the set ω(usually written $\aleph_0$) is the first infinite *cardinal* number. But when the *real* numbers are extended by extrapolation to include one or more infinite numbers one needs to do it in a way that fits in with the rules for multiplying by real numbers. Usually one adds extra numbers at both ends denoted by ∞ and $-\infty$, with rules that allow only a limited extension of arithmetic to these new numbers (expressions like $1/\infty$, for instance, are not allowed). There is, however, an alternative option of using *non-standard numbers* which just keep on going with a (very long) lineful of (nameless) infinite numbers obeying precisely the same rules as the finite real numbers. In this case, if *K* is an infinite number then $1/K$ is a well defined *infinitessimal number*.

4.2 Ideas related to infinity

4.2.1 Infinity in Matte Blanco

Matte Blanco's argument for the role of infinite sets appears most succinctly on pp 146-7. "We may interpret [II_2] as the expression of the fact that the part has the same power or cardinal number as that of the whole. If we now apply the definition [of an infinite set] of Dedekind, this would mean that … we are in fact confronted by an infinite set." The problem with this argument is that it is drawing on a very specific aspect of set theory and applying it to bilogic, despite the fact that set theory, as we have seen, models bilogic rather badly. Moreover (as I shall discuss shortly), Matte Blanco goes on to identify this particular notion of infinite cardinality with other aspects of infinity arising in other number systems. In fact the set theory definition of infinite cardinality fits very badly with bilogic. In bilogic *all* subsets are equivalent to the whole, whereas in set theory not all subsets have the same cardinality as the whole. Indeed, in bilogic even an *individual* is equivalent to the whole, and an individual presumably has cardinality 1, which is not infinite!

To understand more closely the real role of the infinite in bilogic, which I claim does not have to do with infinite cardinality, we need to turn to other arguments of Matte Blanco that suggest a different concept. On p 157 he writes, rather cryptically:

> "We may mention here some examples of classes or sets which belong to the type usually seen in (symmetrical) unconscious thinking:
>
> - the class of good people
> - the class of bad people
> - …
>
> A consideration of these soon shows that in all such cases the values of *x* can be, conceptually, an infinite number. Moreover, the condition *y* specified is itself a variable. One can, for instance, be good, very good, extremely good, etc."

What he has in mind here seems to be the idea that in the class of good people, for example, there is a quality of goodness which is *ordered,* and that good people are those whose goodness is greater than some limiting value – let us designate that value by the symbol "0" – below which people can no longer properly be called good (and, if they fall too far below it, may start being somewhat bad). Thus in set theory terms Matte Blanco seems to be considering the definition

$$\text{Good\_people} := \{ x \mid (x \text{ is a person}) \,\&\, (\exists y)(\text{goodness}(x) = y \,\&\, y > 0) \}$$

or in words

"Good_people" is the set of all people *x* whose goodness is *y* with *y* greater than 0

Matte Blanco, in the quotation above, is then saying that for each degree of goodness *y* in the above definition there may be, conceptually, an infinite number of people *x* that have that goodness, and in addition there are an infinite number of possible degrees of goodness *y*. In his words, since "these conditions (*y, q, z* etc.) … can be conceived as being susceptible to assuming an infinite number of values which we may order in series of ascending magnitude" and "Each of these conditions (*y, q, z* etc.) may, in its turn, be assumed by an infinite number of individuals … we find that the unconscious deals with the type of classes which contain at least one, preferably several, infinities."

At first glance this way of thinking about it seems unnecessarily complicated, since one could more simply rewrite the definition as

> Good_people := { *x* | (*x* is a person) & (goodness(*x*) > 0) }

in words

> "Good_people" is the set of all people *x* whose goodness is greater than 0

with no reference to *y* or to "several infinities". The point is, however, that in actual clinical experience a crucial role is played by the fact that there is an *ordered* series of goodness-degrees at the heart of this definition, and it is the infinity associated with the ordering, rather than the infinity associated with the number of *x*s for a fixed *y,* that is particularly significant, because it brings in the notion of *extremity* or *maximality*.

Consider the case history presented on p 166 where the patient

> "attributed to [the man she had loved] all the good features which she attributed to the class [of fathers] and which do not necessarily have to be possessed by every one of its elements. In other words, she had identified him with the class, and the latter was conceived in her unconscious as having in a maximum degree the characteristics that define it. … [but subsequently] he no longer represented to her the infinite magnitudes which are implied by the propositional function corresponding to this class."

I suggest that it is this conception of "infinite magnitude" of an ordered characteristic that is the most important meaning of "infinity" in bilogic, rather than the rather abstract Dedekind definition of infinite cardinality. In other words, the *infinity we are dealing with here is nothing to do with the possibility or impossibility of putting one subset into one-to-one correspondence with another, but with a feeling of vastness or boundlessness.*

We can usefully relate this to the alternative approach to bilogic through interacting cognitive subsystems. This feeling of vastness is a characteristic emotional and conceptual part of cognition through the implicational subsystem. It is Blake's "Hold infinity in the palm of your hand/ And eternity in an hour"[8], or the infinite revelation of Nicolas of Cusa (to whom I return in the next section), who, in his *De Visione Dei* 53 (on the vision of God) cries out "My God, you are absolute infinity itself … the infinite end." In this mode of cognition, the identification of the member of an *ordered set* (a particular degree of a quality such as goodness) with the whole (all degrees of goodness) leads one from the finite to the unbounded, and leads one to include the whole set of all degrees as the ultimate extreme of the quality of goodness.

Mathematically there are two points to note here. The first is that the attribute of boundlessness of a set of magnitudes is not directly to do with cardinals, but is connected with various kinds of measurability, asserting that the there is no limit to the size of a magnitude. One can have a set of infinite cardinality without its being unbounded. Second, the notion of adding the whole unbounded set of magnitudes as an extreme element of the original set of finite magnitudes is a familiar operation in mathematics: in a literal sense it is the way in which one forms the first infinite *ordinal* number from the finite natural numbers. From a more formal viewpoint, it is also an example of the incorporation by extrapolation of the ideal number ∞ into the real numbers. In Appendix 2 I link these ideas and describe how one can apply this process of extrapolation to any ordered set, in such a way that the entire set itself becomes the ideal infinite element. This construction introduces the entire set as the (transcendental) *maximum* of the set, and thus the equality in symmetric logic between a member of a set and the set itself gives rise, in the case of an ordered set, to an identity between a member of an ordered set and the maximum of the ordered set.

When we come to formulate bilogic more formally later, this principle will be restated in a way that does not lean on set theory to the same extent. The above account is in fact a set theoretic "halfway house" between Matte Blanco's formulation and a more homogeneous non-set theoretic version which I will present.

4.2.2 The coincidence of opposites

Any ordered set has a second ordering which is obtained simply be reversing the first. The set of degrees of moral qualities can be ordered by goodness or by badness. Thus just as a moral quality can, in bilogic, be identified with the maximum of goodness, by referring to the opposite ordering it can equally be identified with the opposite, the maximum of badness. Total good and total evil thus become identified. Many authors (e.g. Chadwick, 2001, p 88) have noted this remarkable, but highly characteristic phenomenon. Expressed generally, within an ordered set the identification between the individual and the whole leads on, via the original and the opposite orderings, to an identification with both the maximum ($+\infty$) and the minimum ($-\infty$) of the ordering.

The tendency to proceed to an infinite magnitude is particularly characteristic of the thought of mystics, which Bomford [2] has convincingly argued is also based on symmetric logic. Moreover, in many cases of mystical experience the two extremes are identified. Nicholas of Cusa, for example, writes "The simply and absolutely maximum, than which there cannot be anything greater, is greater than we can comprehend … And just as there cannot be a greater, so for the same reason there cannot be a lesser … But the minimum is that than which there cannot be a lesser. Because the maximum is of this sort, it is obvious that the minimum coincides with the maximum" (De docta ignorantia, 11). He goes on to develop this into a general principle of the coincidence of contradictories:

"... I have discovered that the place where you are found unveiled is girded about with the coincidence of contradictories. This is the wall of paradise, and it is there in paradise that you reside. The wall's gate is guarded by the highest spirit of reason, and unless it is overpowered, the way in will not lie open." (De visione dei, 37)

This means that for (partially) *ordered* sets we do have a version of Bomford's law of negation. The assertion of any ordered quality, such as "John is powerful", becomes the identity of John with infinite power, and this in turn with powerlessness.

5. The structure of bilogic

5.1 Outline

We now have all the ideas needed to put together a possible mathematical formulation of bilogic which is consistent when considered as a mathematical model, even though its application within any given practical context produces results which are inconsistent according to ordinary logic. I will first summarise these ideas from the previous sections.

1. The logic is context dependent (2.2.1)
2. It is composed of two distinct parts, the symmetric and the asymmetric (3)
3. The symmetric part:

   a. requires a level of description that does not differentiate between members and subclasses (2.2.1, 2.2.2)
   b. does not admit the operation of negation (2.2.3)
   c. does not have a principle of contradiction (2.2.3)
   d. is based on a formal language that ensures that relations are automatically symmetric (2.2.4)
   e. uses one-term relations to allow an ambiguous role to the element "me" (2.2.5)
4. Ordered sets, when viewed from the perspective of the symmetric part, have an infinite maximum element (4.2.1) and a coinciding infinite minimum element (4.2.2)

In this section I will give a brief and general overview of a proposal for the formal specification of bilogic. Further details are given in Appendix 1 and Appendix 3.

5.2 The concept of context dependent logic

To begin with, I want to return to the question, raised at the outset in section 1.2, of what we are to mean by "logic", this time in relation to the formal content of logic, rather than to its application.

The problem with bilogic is that it is apparently inconsistent: it is possible to deduce contradictory statements. In classical logic, this would mean that it would then be possible to deduce all possible statements, and the logic would then be trivial, devoid of any content. My main proposal here is that this can be avoided by regarding symmetric logic (and hence bilogic) as a context dependent logic, in which a changing context in the course of the deductive process prevents this descent into triviality.

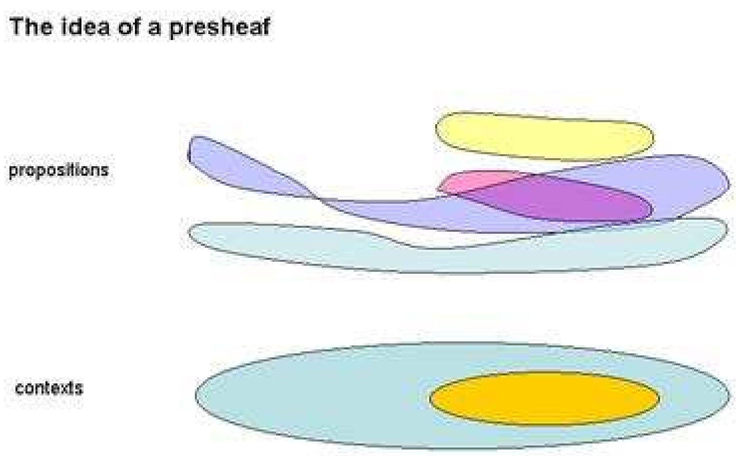
The context dependence of bilogic will be of a more radical kind than that of other non-classical logics that have been considered. Take the example of temporal logic. At this moment I might utter "The cat sits on the mat", and in an hour's time "The cat sits on the window ledge." These are not inconsistent, because the time of utterance is a context that differs between the two statements. This is, however, only a weak sort of context-dependence for two reasons. First, because *within any one temporal context* all allowed derivations can be made without fear of contradiction, and second because by incorporating the context in the propositions (e.g. "The cat sits on the mat at 2.30 pm 10/11/2002") we can work with propositions that are absolutely true or false. In bilogic, by contrast, the process of deduction is linked to a change in context: any deduction produces a new proposition which holds in a different context from the previous ones. Deduction becomes a part of a dynamical system, not an unfolding of an eternal verity.

The situation is similar to, but more radical than, the replacement of Newtonian physics by Quantum physics. In the former, it was similarly assumed that there was an ultimate level of precision, an ultimate context, to which everything could be reduced, namely the positions of all the particles in the universe at all times, while in the latter there is no such ultimate context. For this reason, there are formal connections between quantum logic and bilogic.

Matte Blanco's own hints as to how context dependence might be formalised are remarkably prescient of modern developments in logic. On pages 51-52 he speculates that the breakdown of the principle of contradiction might be imagined by analogy with a three-dimensional space which contains an infinite number of planes. On one plane a proposition *p* might hold, whereas on another the proposition not-*p* might hold; yet all the planes might somehow be coordinated into one logic. This picture of an infinity of planes expresses the essence of the mathematical structure now known as a *sheaf,* which serves as the basis for a very general class of logics, including quantum logic. A sheaf is something like a heap of leaves, all roughly parallel, with the added (and non-visualizable!) properties that nearby leaves can both pass through each other and join smoothly onto one another. Each leaf represents the collection of propositions that hold in a given context. The only difference between this picture and Matte Blanco's account is that in a sheaf there is no prescribed structure (as would be the case for an additional *dimension*) for the leaves that are stacked above a given context.

To be more precise about a sheaf [9] – the leaves are piled up over a *base* which is a topological space: that is, a set having a distinguished collection of subsets called *open subsets* satisfying a set of axioms (such as, that the intersection of two open subsets is another open subset). With each open set *A* there is associated a *set* *S*(*A*) whose elements are called *leaves* or *sections*. For a pair of open sets *A* and *B* such that *A* is a subset of *B* there is a r_{AB} associating each leaf in *S*(*B*) with a leaf in *S*(*A*) (the latter leaves being the "parts" of the leaves over the larger set *B* which lie over the smaller set *A*).

We use this picture (which will shortly be generalised somewhat) to make precise the relationship between context and logic. The open sets represent *contexts,* which may be wider or narrower, expressed by the inclusion of one open set in another, as a subset. The set *S*(*A*) associated with *A* represents the collection of propositions in the bilogic that hold in the context *A.* (See figure below)



Unfortunately this concept is still a bit too restricted for bilogic, mainly because (as just discussed) we need to deal with a dynamic process rather than a static entity. First, context need to be related not just by inclusion but also by succession: we need to represent the idea that one context comes *after* another. This requires the base to be more elaborate than just a topological space. Second, the sets of propositions over different contexts need to be related not by a simple correspondence, but by a process of logical deduction with an appropriate form. The propositions over a context *A* that follows a context *B* are required to be deducible from the propositions over the context *B*. This forces us to consider a generalisation of a sheaf, namely a *functor category.* Details are below in Appendix 3.

5.3 A sketch of bilogic

5.3.1 Overview

As noted in point 2 of 5.1, bilogic will consist of two parts, symmetric and asymmetric (implicational and propositional, respectively). The parts will be based on different formal languages (that is, ways of writing down the *propositions* of the logic). A collection of propositions, from both of the parts, will be called a *text.* Texts are the main objects of the system: each context is associated with a text that defines the cognitive situation in that context. Each part of the logic will have its own rules of inference – ways of deducing new propositions, and hence new texts, from a given text. Note that we choose to present the system in terms of rules of inference, which give us steps in a dynamic process, rather than in terms of axioms, which are normally regarded as a particular set of propositions that are eternally true.

In addition to rules of inference operating *within* each part of the logic, there are rules of inference operating *between* the two parts, enabling one to "translate" from one to the other. But, as always, a great deal is lost in the translation. These rules correspond to Matte Blanco's idea of "unfolding and enfolding" between the two systems of conscious and Unconscious.

5.3.2 The symmetric part

This, the logic of the unconscious, is to be extremely basic. The two fundamental structural relations of set theory, membership ($\in$) and subset ($\subset$), are, following point 3a of 5.1 above, replaced by a single relation $\prec$, pronounced, and meaning "is a". For ease of reading I will often write this as "`isa`". For example, clause 1 in section 1.1 could be written "Peter `isa` dangerous_thing". We introduce a rule of inference which asserts the symmetry of this relation, in the sense that from $A \prec B$ we can deduce $B \prec A$. The relation $\prec$ is a structural part of the system of bilogic, and its symmetry properties are embodied in a rule of inference. Note that this makes the principle of generalisation (section 2.2.2) automatic.

The symmetry of other relations is less fundamental and is embodied (point 3d of 5.1) in the syntax of the language rather than in the rules of inference: it is not possible to write down an asymmetric relation. The syntax used for asserting one of these general relations can be exemplified by

[John, James] `isa` brother

where the square brackets define an *unordered* pair. They can also be used for triples and higher order relationships.

Following 3e of 5.1, the special element "`me`" is a sort of joker which can be added to such pairs, triples etc. Details are given below in Appendix 1.

The rules of inference are the symmetry rule for …, the rules for inserting and removing `me`, and rules for replacing *A* by *B* whenever $B \prec A$.

5.3.3 The asymmetric part

My main argument in this paper has been that set theory is an inappropriate basis for symmetric logic. So, when we come to consider the appropriate formalism for asymmetric logic, we can note that set theory was only introduced by Matte Blanco in an effort to model some of the curious properties of the symmetric realm. In the everyday world of asymmetric logic, we could therefore make do with a more conventional form of classical logic older than set theory. Further grounds for doing this are: first, because our aim is not to find a foundation for mathematics, but to describe the actual processes of thought; and second, because having dropped set theory for symmetric logic, we would only increase the difficulty of matching to it if we were to re-introduce it into asymmetric logic.

There is, however, a very wide choice of possible formalisms open to us. Most of them are geared to a concept of a fixed eternal truth, and designed to explore issues of overall consistency and completeness, rather than modelling the ongoing process of thought. In the Appendix I shall suggest a possible compromise which extracts the main features of conventional logical thinking, while remaining formally rigorous.

The main change from set theory is that the idea of the set is replaced by the idea of the propositional function that defines the set. The two are not equivalent, since there are many propositional functions that could define a given set. It is clear, however, that what is immediately the object of thought is not the set as an assembly of entities, but the abstract defining principle which singles these entities out and assembles them in the first place. Moreover, it is almost invariably the propositional function rather than the set itself that concerns Matte Blanco.

In addition, I do not introduce the quantifiers "for all" and "there is" which play a dominant role in conventional predicate calculus. A constructions involving the first, such as "all humans are mortal", is replaced by the expression "mortal(human)" relating the two propositional functions involved, without reference to any individual. The construction “there is”, asserting an abstract existence without specifying what the thing actually is, is eschewed on the grounds that this is far removed from everyday thought.

To match the way in which *ordered* sets have a special role in symmetric logic, generating infinity in the sense of a maximum, I introduce a special construction corresponding to the English language comparative. The idea is that, instead of defining in abstract terms the idea of a relationship that is an order, we introduce for each quality a concrete order relation corresponding to having this quality to a greater or lesser extent. Of course in some cases (such as the jocular “a teeny bit pregnant”) such a comparison is in practice vacuous, but it is always present as a logical possibility.

Another significant absence from the scheme is anything corresponding to the idea of the equality (“=”) of two sets that happen to be defined by different formulae. In the current scheme, formulae represent statements that are actively derived by an ongoing process, and if the forms of two statements differ, then they may represent stages in different processes and hence cannot be considered as equivalent.

The rules of inference then reconstruct the properties of normal logic on this rather unconventional basis. The details are in the Appendix.

5.3.4 Enfolding and unfolding rules

These are terms used by Matte Blanco (echoing both Nicholas of Cusa and David Bohm) for the rules of inference which “translate” between the symmetric and asymmetric parts. Because the former is much simpler than the latter, a great deal is lost in the process: negation, the ordering of the terms of a relation, and the distinction between membership and subset, are all “washed out” in translating from asymmetric to symmetric form. Conversely, translation in the opposite direction is non-unique: a number of different choices are possible for the asymmetric expression that corresponds to a given symmetric expression, depending on the context and the overall process.

In one respect, however, the symmetric realm is richer than the asymmetric, namely in possessing an experience of infinity arising as a maximum element to an ordered set (point 4 in section 5.1). This makes for a non-uniqueness for translation in the direction of asymmetric to symmetric, in which any attribute possessed by an individual can be promoted to an infinite extent.

6. A comparison with quantum logic

Bilogic and quantum logic can both be formulated as context-dependent logics based on a sheaf-theoretic framework. Doing this in a fairly rigorous manner shows that they have a family resemblance, but – perhaps more importantly – it highlights many detailed differences between the two, which I will briefly survey.

1. The nature of a context differs between the two. In quantum logic, a context is defined precisely by the measurement situation, and it specifies a particular Boolean sub-logic, holding at a given time. In bilogic the idea of contexts is no more than a framework in which to fit a process of developing texts (sets of propositions); there is no clear idea of an explicit formative role for different contexts.
2. The relation between time and logical derivation differs in the two systems. In bilogic, derivation is a process, from one context to another which can be regarded as temporarily later. In quantum logic derivation (the extension of truth values to all propositions in a Boolean lattice, given truth values on a basic subset) is an abstract process of ensuring mathematical consistency at each particular time. Time enters through a dynamical evolution of the context and of the quantum state, as separate factors from the process of logical derivation.
3. The sheaf-theoretic basis differs somewhat. In quantum logic (as in other topos-oriented logics

such as intuitionism) the context are ordered in terms of increasing fineness of measurement; in intuitionism there is a concept of cumulatively increasing stages of truth. The range of propositions asserted in each context increases cumulatively with this ordering. In bilogic there is no such cumulation and no basis for a conventional ordering that can be identified globally as any sort of time (there is no time in the Unconscious). As a result, the sheaf-theoretic approach in bilogic is based on the concept of paths – ways of getting from one text to another – rather than on an overall ordering.

4. Inconsistency is more radical in bilogic. In quantum logic each Boolean algebra has consistent truth values within itself, although they may differ in other algebras over different contexts. In bilogic any text may contain mutually inconsistent propositions.

Quantum logic and bilogic thus occupy respectively conservative and radical ends of the spectrum of non-standard logics. In this way they stake out a new ground for modelling about our world – the world as thought by us – in a very new way.

Apendix1 The formalism of bilogic

A1.1. A specification for symmetric logic

This appendix defines a possible formal language for symmetric logic in a form intended for readers with a philosophical bent but unfamiliar with mathematical conventions. With this readership in mind I begin with some remarks about such conventions. I will call the formal language being defined **SL.** In this appendix, **bold** type is used for emphasis and/or to indicate a term being defined, all other typographic aspects having special meanings.

Normally in writing the marks on the page make up linguistic elements (words, sentences) which **denote** things. The same is the case with mathematical formulae, the linguistic elements being variables, constants, propositions, equations and the like, and the things denoted normally being abstract mathematical entities. Mathematical linguistic elements are conventionally written in italics, to distinguish them from elements of ordinary written language. Here we will be writing formulae (in an informal language) about a formal language SL, and so the things denoted are themselves linguistic elements of SL. In order to refer to a linguistic elements (rather than that which the element denotes) it is placed in inverted commas. Compare the examples:

Let *A* be a finite set

and

"A" is the first letter of the upper case Roman alphabet

Where inverted commas are clumsy or ambiguous, `courier type` will be used for an element of informal descriptive mathematics which denotes itself regarded as an element of SL. I will not be slavishly formal in this account, sometimes relying on context and common sense to indicate what language is being written at any given point.

It is sometimes convenient (or even essential) to distinguish between a particular occurrence of an element (such as the "It" that appears at the start of this sentence) and the general word itself. A particular occurrence is often called a **token** of the element.

In describing the syntax of SL it is convenient to use **syntactic templates**: formulae on the page containing words or symbols that can be replaced by appropriate tokens of linguistic elements so as to provide valid expressions in the language. As an example taken from arithmetic, consider the sentence

Fractions are denoted by "*p/q*" where *p* and *q* are numerals

The inverted commas indicate that we are talking about the linguistic entity they enclose, but the following where-clause specifies that we do not take the symbols "*p*" and "*q*" literally, but replace them by arbitrary numerals to produce things like "3/4", "2/3" etc. One version of this, taken from computer science, is illustrated by

"⟨number1⟩ / ⟨number2⟩"

as a more descriptive alternative to "*p/q*".

Language

I now want to propose a language (which I will call SL) that takes account of some of the key objections raised in this paper to the use of conventional set theory: namely, membership should not be distinguished from subclass, there is no negation, relations are automatically symmetric. The syntax is inspired by the computing language perl (and similar object-oriented languages) where there is a fundamental relation `isa` (pronounced, and meaning "is a") between the types of objects. To say that "*A* `isa` *B*" means that all the attributes of objects conferred by their being of type *B* are also attributes conferred by type *A*. (Example: *dog* `isa` *mammal*). I shall extend this by (a) symmetrising (at the level of the rules of inference); and (b) allowing it to apply to all entities, not just types. To remind us of the origin in Matte Blanco's set theory I will use the symbol "◄" for this concept, but the idea is to regard it as corresponding *both* to "subclass" and to "member".

Syntactic issues

Symbols and strings

The basic linguistic elements of the language SL are **strings**. A token of a string is a linear array of upper and lower case letters, the underscore character, numerals, the punctuation marks

, [ ]

and the sign

◄

(pronounced "is a"). Punctuation marks and "◄" are called special characters and the rest are called ordinary characters. In giving examples I will sometimes informally replace "◄" by "`isa`" for ease of reading.

In a linear array that is a token of a string, spaces are ignored, symbols in different type faces etc are identified, and certain arrays differing only in their ordering as specified below are also identified, being regarded as tokens of the same string; but otherwise ordering matters.

Strings can be concatenated; "⟨string1⟩ ⟨string2⟩" represents the concatenation of the strings "⟨string1⟩" and "⟨string2⟩", formed by concatenating their tokens. If two tokens are such that they are regarded as the same string, then the tokens arising from concatenating them with some other linear array are also to be regarded as the same string.

Names

A name is a non-empty string consisting only of ordinary characters.

Multiplets

A multiplet is a string started by "[" and ended by "]", these enclosing a sequence of one or more names, with consecutive names separated by a single ",". Tokens of multiplets differing only in the order of the names are identified as the same string; e.g. "[⟨name1⟩,⟨name2⟩,⟨name3⟩]" is the same string as "[⟨name3⟩, ⟨name1⟩, ⟨name2⟩]"

Propositions

Propositions are strings of the two following kinds:

(i) An *assertion* is a string of the form

$\langle \text{name1} \rangle \blacktriangleleft \langle \text{name2} \rangle$

(ii) A *relation* is a string of the form

$\langle \text{multiplet} \rangle \blacktriangleleft \langle \text{name} \rangle$

where $\langle \text{multiplet} \rangle$ is any multiplet other than "[`me`]".

In the second of these "relation" ought really to be "relational assertion" but this is a clumsy expression, and there is no ambiguity involved in abbreviating it to "relation" because we do not need to distinguish between classes, individuals and relations at the level of names. The way the notation works is illustrated by the following examples.

[John, Jill] `isa` brother

corresponds to *both* "John is the brother of Jill" and "Jill is the brother of John". Note that contradictions of gender, like all contradictions, are allowed!

John `isa` brother

expresses the idea that John is a brotherly sort of person. The name "brother" is not tied down to be precisely one of either a class or a relation; it is *both.*

[John] `isa` brother

means that "John is my brother". The rules of inference relate the last two examples to each other and to

[John, me] `isa` brother

which also means "John is my brother" but arising in a form where "me" is explicitly symbolised rather than being implicit.

Texts

A text is a finite, unordered, collection of propositions. Thus I will denote a text by an expression of the form "{ *p, q, ..., t* }" where "*p*", "*q*", …, "*t*" are propositions.

Rules of inference

A rule of inference asserts the allowability of producing a proposition *p* from a text *T* = { *q, r, ..., t* } . To express this, I will write

$$q, r, ..., t \longrightarrow p$$

Note that $\longrightarrow$ is not a symbol from the language SL; it expresses a meta-logical relation. I propose the following rules.

1. Symmetry

For any assertion $m \blacktriangleleft n$, where *m* and *n* are names,

$$m \blacktriangleleft n \longrightarrow n \blacktriangleleft m$$

This is the fundamental symmetry property of symmetric logic, the basis of the part being identical to the whole.

2. Transitivity

(i) $s \blacktriangleleft m \,,\, m \blacktriangleleft n \longrightarrow s \blacktriangleleft n$

where *s* is a name or a multiplet and *m, n* are names.

(ii) $m \blacktriangleleft n \,,\; s \blacktriangleleft t \longrightarrow s' \blacktriangleleft t$

where *m, n* are names, *s* is a multiplet, and s' is a multiplet obtained by replacing any occurrence of *n* in *s* by *m*.

Note that if we are given that both $m \prec n$ and $n \prec m$ (the second following anyway from the first, by 1) then these rules allow us to replace any occurrence of *m* by *n,* and vice versa. As a result, we do not need to introduce equality ("=") as part of the language SL, because these rules do the same job. This is an advantage, because equality, the assertion that two different names actually denote the same thing, is too subtle and high-level an assertion to have a place in SL. Equality is implicit in the operations, not the subject of an explicit assertion.

3. The implicit me

(i) $[\,n\,] \prec m \longrightarrow [\,n, \text{me}\,] \prec m$

(ii) $[\,n, \text{me}\,] \prec m \longrightarrow [\,n\,] \prec m$

(iii) $[\,n\,] \prec m \longrightarrow n \prec m$

(iv) $n \prec m \longrightarrow [\,n\,] \prec m$

where in all the above *n* is a name other than "me" and *m* is a name.

This formalises the equivalences discussed under "relations" above.

A1.2 A formalism for asymmetric logic

As noted in the main text, our aim is to find a version of classical logical, rather than set theory, which is a compromise between precision and modelling the actual processes of thought. The result is, of course, much more long-winded than set theory, because a great many propositions that are theorems in set theory have to be put in "by hand" in the current model. And this is quite deliberate: as we have in mind a context dependent logic in which the context changes with each operation of derivation, it is conceptually helpful to introduce a distinction, however rough and to some extent arbitrary, between those deduction which are "automatic" and have the status of rules, from those that are more elaborate and require, in practice, a significant amount of thought.

Language

The language consists of *individual constants,* represented by *a, b, c, …* ; *functional constants* represented by *f, g, …* ; *connectives* $\vee$ $\wedge$ $\sim$ || ; the comparator $\rightarrow$; comma ; and the brackets ( ) $\langle$ $\rangle$

A *multiplet* is a string of the form $\langle$p, q, … , r$\rangle$ where "p, q, … , r" denotes a comma separated list of constants, either all individual constants or all functional constants. All brackets can be omitted where there is no ambiguity

Each functional constant is associated with a *domain* consisting of constants and

multiplets

Syntax

An *elementary functional expression* is either a string of the form *f*(*z*) where *f* is a functional constant and *z* is a string in the domain of *f,* or a string of the form $f^{\rightarrow}(\langle w, z\rangle)$ with both *f*(*w*) and *f*(*z*) being elementary functional expressions.

A *functional expression* is either an elementary functional expression or a string of the form $(q \cup f)$ (z) with both $(f)(z)$ and $(q)(z)$ being functional expressions.

An *elementary proposition* is either a string (z) with z a functional expression or an expression $(f \,||\, g)$ with f and g functional constants.

A *proposition* is either an elementary proposition or one of the following

$$(\varphi \vee \psi) \qquad (\varphi \wedge \psi) \qquad \sim(\psi)$$

with φ and ψ propositions.

Examples

English	**formal language**	**instance of …**
Socrates, Glauco		individual constants
man, human, wise, mortal		functional constants
Socrates is a man	man (Socrates)	(elementary) functional expression (and proposition, ignoring brackets)
All humans are mortal	mortal(human)	(elementary) functional expression/proposition
Socrates is wiser than Glauco	wise$^{\rightarrow}$(⟨ Socrates, Glauco⟩)	(elementary) functional expression / proposition
Ash trees, gooseberry bushes	ash_tree gooseberry_bush	functional constants
Ash trees grow taller than gooseberry bushes	Grow_tall$^{\rightarrow}$(⟨ ash_tree, gooseberry_bush⟩)	(elementary) functional expression /proposition
Humans are either men or women	(man $\cup$ woman) (human)	non-elementary functional expression / proposition
no one is both a man or a woman	man || woman	elementary proposition
Either this apple is mouldy or I'm a dutchman	mouldy(this_apple) $\vee$ dutchman(me)	non-elementary proposition

Rules of inference

Specimen rules of inference are then of the following form, where I have not given an exhaustive list, but tried to indicate the principles that cover all the essential ingredients of logic. (There is in fact considerable freedom in choosing what might constitute a complete list) As in the previous section, the braces { } have been omitted from texts.

The text …		**allows the deduction of …**
Rules of substitution		
$f(g)$, $g(x)$	$\rightarrow$	$f(x)$
$p(\langle g, h, \ldots \rangle)$, $g(x)$, $h(y)$ …	$\rightarrow$	$p(\langle x, y, \ldots \rangle)$
Rules governing order		
$f^{\rightarrow}(\langle w, z \rangle)$, $f^{\rightarrow}(\langle z, q \rangle)$	$\rightarrow$	$f^{\rightarrow}(\langle w, q \rangle)$
Rules linking operators at different levels		
$(q \cup f)(z)$	$\rightarrow$	$q(z) \vee f(z)$
$q(z) \vee f(z)$	$\rightarrow$	$(q \cup f)(z)$
$q(z)$, $f(z)$	$\rightarrow$	$q(z) \wedge f(z)$
$q(z) \wedge f(z)$	$\rightarrow$	$q(z)$
$q(z) \wedge f(z)$	$\rightarrow$	$f(z)$

(The usual associative rules can be added)

Rules involving negation		
$f \parallel g$, $f(x)$	$\rightarrow$	$\sim g(x)$
$(q \cup g)(x)$, $\sim g(x)$	$\rightarrow$	$q(x)$

(In the above, p is a functional constant or the comparative $f^{\rightarrow}$)

A1.3 Unfolding/enfolding rules

Having restructured classical logic, we find that there is less difference between the asymmetric and symmetric components as might appear. By disposing of the set theoretic underpinning, the only distinction between individuals (which enter into membership relations) and functional constants (the analogue of classes, which enter into subset relations) is that the expression $f(x)$ is defined only if f is a functional constant; in other words, an individual is a function whose domain happens to be empty. This makes the formulation of these rules of translation more straightforward than would otherwise be the case.

The list below is not exhaustive, but is intended to illustrate the general principles that would be involved in any formulation

From the text …		**one can infer …**
$f(x)$	$\rightarrow$	$x \prec f$

$x \prec f$	$\rightarrow$	$f(x)$ provided x is in the domain of f (this proviso is always understood in the following)
$f(x, y)$	$\rightarrow$	$[x, y] \prec f$
$[x, y] \prec f$	$\rightarrow$	$f(x, y)$
$[x, y] \prec f$	$\rightarrow$	$f(y, x)$

The following is a rule-schema , rather than a single rule:

If φ is a proposition in asymmetric logic, and ψ a proposition in symmetric logic, such that $\varphi \longrightarrow \psi$ then $\varphi \longrightarrow \sim \psi$. It should be interpreted in the sense that, for every rule of deduction from asymmetric to symmetric there is a corresponding negated rule.

In order to understand the role of order and the maximum we need to note a distinction between the meaning of functional constants representing qualities in the symmetric and asymmetric components of bilogic. For example, the asymmetric adjective "red" can pass into the symmetric noun "redness"; from "this ball is red" we can obtain "this ball is redness". In terms of the set theoretic treatment of infinite degrees above, "redness" is the entire class of degrees of red, the class becoming the maximum of the set. If we denote this maximal noun corresponding to a quality f by $f*$, then we have the inferences

$f^{\rightarrow}(x, y)$	$\rightarrow$	$x \prec f^*$
$x \prec f^*,\ y \prec f$	$\rightarrow$	$f^{\rightarrow}(x, y)$

Appendix 2 – ideal infinities

In this appendix I describe more fully the idea of constructing an ideal maximum for an ordered set. Let us denote the set in question by A and the ordering by $\leq$. In one sense the construction is trivial. One takes an arbitrary element which we name ∞, append it to A, thereby defining $A^* := A \cup \{ \infty \}$, and extend the order to A^* by simply defining a new order $\leq^*$ which is identical with $\leq$ on A, but in addition

for any $y \in A$, $y \leq \infty$; and

$\infty \leq \infty$

While this purely formal procedure is entirely normal in mathematics, it is instructive for our purposes to be more specific about the construction of this ideal element ∞. To this end, I want to digress in order to describe in more detail the construction of the Ordinal numbers, with the intention of using that construction to perform an analogous one in a general ordered set.

Ordinal numbers (or "ordinals" for short) have a rather neat definition in set theory: each ordinal (from 0 onwards) is defined to be the set consisting of all the smaller ordinals. Thus the ordinal 0 is the set with no members (the empty set) because there are no ordinal numbers smaller than 0, 1 is the set whose only member is 0, and so on. This means that 0 has 0 members, 1 has 1 member and so on. The set ω, defined as consisting of all the finite numbers, is then one of the ordinals, in fact the first infinite ordinal. One can continue the process from there on, defining $\omega + 1$ to be the set $\{0, 1, 2, \ldots, \omega\}$, and so on, though we will not require this here. Note in passing that $\omega + 1$ can be paired off with the elements of ω according to the scheme

$0 \leftrightarrow \omega,\quad 1 \leftrightarrow 0,\quad 2 \leftrightarrow 1,\quad 3 \leftrightarrow 2,\quad 4 \leftrightarrow 3,\quad \ldots$

so that ω and $\omega + 1$ have the same cardinality, though they are different ordinals. This illustrates the fundamental distinction between infinity as a cardinal and infinity as an ordinal.

Based on this, we make the following notational conventions. Given any ordered set A, not necessarily an ordinal number, we define ∞_A to be equal to A, and define A^* to be the set $A \cup \{ \infty_A \}$, proceeding to extend the order to A^* as considered above. The previous construction of the ordinals follows this scheme, with $\infty_\omega = \omega$ and $\omega^* = \omega + 1$. This construction introduces the entire set as the *maximum* of the set, and thus the equality in symmetric logic between a member of a set and the set itself gives rise, in the case of an ordered set, to an identity between a member of an ordered set and the maximum of the ordered set. This is expressed via the unfolding function at the end of Appendix 1.

Appendix 3 - Topos theory approach to logic

This appendix, written for mathematicians with some knowledge of category theory, indicates what I believe is a valid formal context for bilogic.

The rules of inference for the formal logic **L** discussed in Appendix 1 will now be treated purely abstractly as a collection I of pairs (r, s) where r is a set of propositions (usually having only one member) and s is a proposition that can be inferred from r. I is specified by a finite rule-scheme, as in Appendix 1, but it contains an infinite number of possible concrete instances conforming to the rule-scheme. For completeness it will contain the rule that any proposition implies itself. We will denote by P the power set of the propositions of **L,** i.e. P is the set of all sets of propositions. Then the rules of inference I specify in turn a subset $\mathscr{I}$ of $P \times P$ consisting of all pairs (u, v) such that for each proposition s in v there is a subset r of u with (r, s) in I. In this case v is deducible from u and we write $u \Longrightarrow v$.

In bilogic the aim is to depict an ongoing process involving both the “derivation” and the “forgetting” of propositions, governed by a context that is continually changing, with no sense of cumulation. Contradictory propositions can appear in different contexts or in the same context, without the process becoming trivial.

A system of contexts can (in a fuller model than that given here) shape this ongoing process of derivations. By “context” I mean a specification of the occasion on which a thought is formulated or expressed. Contexts are related both by time, when one wishes to describe a succession of thoughts related by the laws of derivation; and also by inclusion, when one wishes to compare the (hypothetical) results of expressing a thought in a wider or narrower context, at a given temporal location. In general I prefer the phrase “temporal location” to “time” or “instant”, to indicate that we may be wanting a looser structure than a conventional linearly ordered time.

In order to express this double relation between contexts, we need both a set K of contexts with a relation $\Rightarrow$ interpreted as succession ($a \Rightarrow b$ means that b is a successor of a) and a relation $\subset$ interpreted as inclusion. For definiteness, we assume that succession is non-reflexive (i.e. for any $u \subset K$, $\neg(u \Rightarrow u)$), but in general it will not be transitive, unless it corresponds to an actual time-ordering. The relations $\Rightarrow$ and $\subset$ are exclusive (they cannot both hold for the same pair of contexts), because inclusion only holds for contexts that are *not* successive.

The basic structures of bilogic are functions f from K to P, interpreted as saying that $f(a)$ is the set of propositions holding in context a. Explicitly, we define a *bilogic function* to be a function $f: K \longrightarrow P$ such that inclusion of contexts corresponds to inclusion of sets of propositions, and succession of contexts corresponds to derivation. That is,

If $a \subset b$ then $f(a) \subset f(b)$ (1)

and if $a \Rightarrow b$ then $f(a) \Longrightarrow f(b)$. (2)

The idea of a function that associates with each context the set of propositions that are to hold in that context is familiar in intuitionism (where the contexts are called “stages of truth”) and in the work on quantum mechanics by Butterfield and Isham where the context is defined by a particular “observable”. As noted in section 6, however, in bilogic we cannot assume an ordering of contexts, and the ordering of texts by inclusion is not appropriate. The structure corresponding to the orderings used in quantum theory and intuitionism is the set of *paths* between contexts and texts, which reflects the process nature of bilogic. To make K and P into categories we shall define a set of *arrows* [10] ($\longrightarrow$) on each of them, where $k : a \longrightarrow b$ means that a and b can be connected by a path k consisting of a succession of (for K) relations $\Rightarrow$ and $\subset$, with a similar construction for P. An operation of composing arrows will then be defined so that f extends to be a functor.

The details are as follows.

First, we extend f to the function $f^* = f \times \mathrm{id} : K \longrightarrow P \times K$. On $P^* := P \times K$ we extend the relations $\Longrightarrow$ and $\subset$ by redefining them as ($\Longrightarrow \times \Rightarrow$) and ($\subset \times \subset$), respectively.

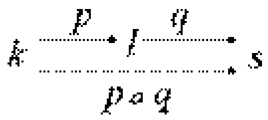
Next, we define a *path* on K to be a sequence $(k_1, k_2, \dots, k_n; l)$ $(n > 0)$ such that

1. when $n > 1$, $k_{i-1} \Rightarrow k_i$ for $i = 2, 3, \dots$
2. $k_n \subset l$.

Composition of paths is then defined by

$$(k, k_2, \dots, k_n; l) \circ (l, l_2, \dots, l_m; s) = (k, k_2, \dots, k_n, l_2, \dots, l_m; s) .$$

Then K becomes a category with the paths as arrows, the path $(k, k_2, \dots, k_n; l)$ having domain k, and codomain l, composition being defined from left to right:



On P^* we similarly define a path to be a sequence $(a_1, a_2, \dots, a_n; b)$ $(n > 0)$ such that

1. when $n > 1$, $a_{i-1} \Rightarrow a_i$ for $i = 2, 3, \dots$
2. $a_n \subset l$.

Composition of paths on P^* is then defined by the two following cases:

a. $(a, a_2, \dots, a_n; b) \circ (b; s) = (a, a_2, \dots, a_n; s)$.

b. $(a, a_2, \dots, a_n; b) \circ (b, b_2, \dots, b_m; s) = (a, a_2, \dots, a_n, b'_2, \dots, b'_m; s)$

where the $b'_2, \dots, b'_m$ are defined inductively according to

$b'_2 = \zeta(b_2, a_n)$

$b'_k = \zeta(b_k, b'_{k-1})$ $(k = 3, \dots\}$

using

$\zeta(b, a) := \bigcap \{ e \mid e \subset b, \& \ a \Rightarrow e \}$

which again produces a category with paths as arrows.

f^* can then be extended to a functor by defining its action on arrows inductively by

$f^*((k ; l)) = (f^*(k); f^*(l))$

$f^*((k_1, k_2, \dots, k_n; l)) = f^*((k_1, \zeta(k_2, k_1); k_2)) \circ f^*((k_2, \dots, k_n; l))$

from which it is clear that the functorial property is satisfied.

Any functor between these path categories will, from the definition given, be derivable from a function satisfying (1) and (2) above, and hence the functor category The set of all functors f^* for some f becomes a *functor* between categories, and the set of all such maps becomes a functor category P^{*K} which serves as a model for bilogic. Although it is not itself a topos, it can be embedded as a sub-catgory of sets and hence provided with a sub-object classifier, which is, however, not itself a member of P^{*K} .

Another way of expressing this is to note that (with a notational change in the direction of one of the arrows) the functions f^* form a *presheaf,* which precisely implements the multi-dimensional picture of Matte Blanco discussed earlier.

One final structural ingredient needs discussing. As so far defined, there is not restriction on the range of propositions that can appear in a given context: because inclusion is an allowed part of the derivation process, new propositions do not have to be derived from proceeding ones. Of course, it is essential that there be some way in which new information can enter the system, otherwise no process will ever get started. However, it might be desirable to control the access of such information in some way. One approach to this is to introduce the empty set as an initial context, prior to all non-trivial contexts, with the empty set of propositions corresponding to it, and to add a rule of inference that any *single* proposition is "inferable" from the empty set. With this controlled way of introducing propositions one at a time we can then restrict any further incursions be requiring that each context be determined by inference from its predecessors, in the sense that, for any context k, the diagram formed from all arrows in $f^*(K)$ having $f^*(k)$ as codomain has $f^*(k)$ itself as co-limit.

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[1] A possible exception to this conclusion concerns pp 227ff, which are inspired by William James and show a tension between the latter's concept of consciousness as awareness and the propositional conception of consciousness earlier in the book. It is possible that Matte Blanco was unaware of the distinction between the two usages.

[2] The essential intension of this axiom is that one cannot have an infinite sequence (i.e. with no first member) of sets each one being a member of the next. Interestingly, its usual formal statement asserts that every nonempty set is disjoint from one of its elements, which is the exact opposite of $II_{2\ a}$

[3] As Matte Blanco says, "the unconscious … knows no individuals, but only classes" p. 181"

[4] The formal system that is the basis of set theory

[5] We should note that 3 does not necessarily imply 2: in logics where truth is context-dependent, the context of a deduced statement may not be the same as the context of the original statement, and thus their truth values may differ.

[6] The same idea occurs also in the theory of Interacting Cognitive Subsystems, treated in the next section, where the underlying subject matter of all items of memory in the implicational subsystem is the way in which they impinge, for good or ill, on the self.

[7] The following account is summarised from Clarke …

[8] *Auguries of Innocence,* 1. 1 (1803)

[9] Strictly speaking, what is described here is a pre-sheaf, which is more general than a sheaf, restricted to the topological version which is closest to Matte Blanco's picture.

[10] This use of the arrow is different from that in Appendix 1